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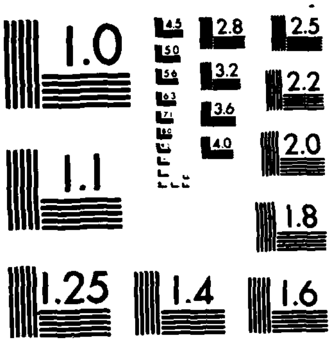


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**20. ABSTRACT (Continue on reverse side if necessary and identify by block number)**

The research centered on the following four classes of "generalized" linear systems:

(1) Linear continuous-time and discrete-time systems whose coefficients depend on (a priori) unknown parameters;

[illegible] systems;

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20. ABSTRACT (continued)

(3) linear continuous-time systems with commensurate or non-commensurate time delays;

(4) linear time-varying discrete-time systems.

Much of the effort was devoted to the study of control for the classes (1), (2), and (3). For the class (4), the primary problem of interest was realization. A brief description of the completed work is given in this report. A complete description of this work can be found in the list of papers in Section 2 of this report.

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CONTROL OF PARAMETER-DEPENDENT SYSTEMS,
SPATIALLY-DISTRIBUTED SYSTEMS, AND SYSTEMS WITH DELAYS

by

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## 1.0 Summary of Results

This ~~The~~ research centered on the following four classes of "generalized" linear systems:

(1) Linear continuous-time and discrete-time systems whose coefficients depend on (a priori) unknown parameters;

(2) linear spatially-distributed continuous-time and discrete-time systems;

(3) linear continuous-time systems with commensurate or non-commensurate time delays; and

(4) linear time-varying discrete-time systems.

Much of the effort was devoted to the study of control for the classes (1), (2), and (3). For the class (4), the primary problem of interest was realization. A brief description of the completed work is given below. A complete description of this work can be found in the list of papers in Section 2 of this report.

### 1.1 Parameter-Dependent Systems

Consider the linear continuous-time system given by the state model

$$\frac{dx(t)}{dt} = F(p)x(t) + G(p)u(t) + Pd(t)$$

$$y(t) = H(p)x(t) + J(p)u(t) + Qd(t),$$

where $x(t)$, $u(t)$, $d(t)$, $y(t)$ are the state, control, disturbance, and output, respectively (all signals are vector-valued). The coefficient matrices $F(p)$, $G(p)$, $H(p)$, $J(p)$ are continuous functions of a parameter vector $p$ which takes its values from a subset $\Omega$ of $\mathbb{R}^N$ (N-dimensional Euclidean space). The particular value of the parameter vector $p$ is <u>not</u> known before the system is in operation, but we assume that $p$ can be estimated on line (during system operation).

Systems whose coefficients depend on parameters arise in applications where one or more of the system coefficients are sensitive to operating conditions. For example, in aircraft control problems, some of the system coefficients may be functions of drag and/or altitude. Systems depending on parameters often result from the linearization of nonlinear systems with respect to nominal operating points that are described in terms of a set of parameters. An example is the satellite problem which is linearized with respect to a nominal radius and nominal angular velocity. Dependency on parameters may also be a result of the application of disturbance signals containing unknown parameters, such as sinusoidal disturbances whose frequencies are not known a priori (before the system is in operation).

We have derived a number of new results on the existence and construction of a feedback system

$$\frac{dw(t)}{dt} = A(p)w(t) + B(p)y(t)$$

$$u(t) = - C(p)w(t) - D(p)y(t),$$

such that the closed-loop system is asymptotically stable for all parameter values $p \in \Omega$, and such that tracking and/or disturbance rejection is achieved for all parameter values $p \in \Omega$. That is, $y(t) \to y_{ref}(t)$ as $t \to \infty$ where $y_{ref}(t)$ is a given reference signal, and the affect of the disturbance $d(t)$ on the output $y(t)$ "dies out" as $t \to \infty$. (See [2], [4], and [9].) Since we are assuming that the parameter vector $p$ can be estimated on-line, the feedback compensator given above is implemented by evaluating the coefficient matrices $A(p)$, $B(p)$, $C(p)$, $D(p)$ in real time. Our approach to the design of the feedback compensator is based on the stabilization of an augmented system consisting of the original system and models for the reference and disturbance signals. Current work deals with the issue of sensitivity of the control process to errors in the estimation of the parameter vector $p$.

### 1.2 Linear Spatially-Distributed Systems

A large part of the research dealt with a class of spatially-distributed continuous-time systems given by state equations with coefficients in the commutative algebra $\ell^1(\mathbb{Z}, \mathbb{R})$ of absolutely summable functions from the integers $\mathbb{Z}$ into the reals $\mathbb{R}$. In particular, a quadruple $(F, G, H, J)$ of matrices with entries in $\ell^1(\mathbb{Z}, \mathbb{R})$ defines a spatially-distributed (or interconnected) continuous-time system given by the dynamical equations

$$\frac{dx(t,r)}{dt} = \sum_{j=-\infty}^{\infty} F(r - j)x(t, j) + \sum_{j=-\infty}^{\infty} G(r - j)u(t, j)$$

$$y(t, r) = \sum_{j=-\infty}^{\infty} H(r - j)x(t, j) + \sum_{j=-\infty}^{\infty} J(r - j)u(t, j), \; r \in \mathbb{Z}.$$

Here $x(t, r)$ is the state at time $t$ and point $r \in \mathbb{Z}$, $u(t, r)$ is the input at time $t$ and point $r$, and $y(t, r)$ is the output at time $t$ and point $r$.

Spatially-distributed systems of the form given above arise in the study of long strings of coupled systems, such as strings of vehicles. They also result from the spatial discretization of partial differential equations which are functions of time and one spatial variable. Such a discretization has been employed (see [6], [11]) in the study of a long seismic cable used in offshore oil exploration.

We have derived a number of new results on the stabilization and regulation of spatially-distributed continuous-time and discrete-time systems. For details on this work, please refer to [6], [11]. We should also point out that the techniques developed in this research can be extended to linear spatially-distributed systems with a two-dimensional discrete spatial dependence. Such systems are actually three dimensional since there is one time variable and two spatial variables. Continuous-time systems with a two-dimensional discrete spatial dependence may result from the spatial discretization of partial-differential equation models for systems that are distributed in a plane, such as large platforms in free space.

## 1.3 Systems with Time Delays

In this part of the work, we studied feedback control of linear neutral (and retarded) time-delay systems with one or more noncommensurate time delays. A new (algebraic) notion of stability, called pointwise stability, was defined and was shown to be generically equivalent to uniform asymptotic stability independent of delay. Necessary and sufficient conditions were derived for regulability, that is, for the existence of a dynamic output feedback compensator with pure delays such that the closed-loop system is internally pointwise stable (and thus stable independent of delay). Necessary and sufficient conditions involving matrix-fraction descriptions were given for the existence of a state realization which is regulable. The problem of stabilization using nondynamic state feedback was also considered in the case when the system's input matrix has constant rank. For details on this work, please refer to [1], [5], [8].

### 1.4 Linear Time-Varying Discrete-Time Systems

The last major research area dealt with the problem of realization in the linear time-varying discrete-time case. A brief description of this problem is given below.

Let $W(k, r)$ be a matrix function of two discrete variables $k$ and $r$, and consider the linear time-varying system specified by the input/output relationship

$$y(k) = \sum_{j=-\infty}^{\infty} W(k, j)u(j).$$

The system with this input/output relationship is said to be realizable if there exist matrix functions $F(k)$, $G(k)$, $H(k)$, $J(k)$ such that the system has the state-space representation

$$x(k + 1) = F(k)x(k) + G(k)u(k)$$

$$y(k) = H(k)x(k) + J(k)u(k).$$

The problem of realization is a very important part of the problem of model building. The time-varying case is of interest since many systems appearing in applications have time-varying parameters. In addition, models for nonstationary signal processes must necessarily be time varying.

Despite the interest in the problem of realization in the time-varying discrete-time case, until this work there were no necessary and sufficient conditions for the existence of a realization. This aspect of the problem has now been completely solved. In addition, we have results on the construction of realizations (state models) with various desirable properties such as minimality. A thorough treatment of the problem of realization in the time-varying discrete-time case will appear in the Ph.D. thesis of Ferrer (the work should appear in August 1984).

## 2.0 List of Research Papers Prepared under Grant


1. E. W. Kamen, Linear systems with commensurate time delays: Stability and stabilization independent of delay," IEEE Transactions on Automatic Control, Vol. AC-27, pp. 367-375, April 1982.

2. E. W. Kamen and P. P. Khargonekar, "Regulator design for linear systems whose coefficients depend on parameters," in Proc. Fourth Meeting of the Coordinating Group on Modern Control Theory, Rochester, Michigan, pp. 359-366, October 1982.

3. W. L. Green and E. W. Kamen, "A local approach to the stabilization of linear systems defined over non-commutative normed *-algebras," in Proc. 22nd IEEE Conference on Decision and Control, San Antonio, Texas, pp. 837-840, December 1983.

4. E. W. Kamen and P. P. Khargonekar, "On the control of linear systems whose coefficients are functions of parameters," IEEE Transactions on Automatic Control, Vol. AC-29, January 1984 (to appear).

5. E. W. Kamen, P. P. Khargonekar, and A. Tannenbaum, "Pointwise stability and feedback control of linear systems with noncommensurate time delays," accepted for publication in ACTA Applicandae Mathematicae.

6. W. L. Green and E. W. Kamen, "Stabilizability of linear systems over a commutative normed algebra with applications to spatially-distributed and parameter-dependent systems," revised version submitted (November 1983) to SIAM Journal on Control and Optimization.

7. W. L. Green, "Stabilizability of systems defined over a commutative normed algebra," accepted for publication in Numerical Functional Analysis and Optimization.

8. E. W. Kamen, P. P. Khargonekar, and A. Tannenbaum, "A local theory of linear systems with noncommensurate time delays," to appear in Proc. 1983 Symposium on the Math. Theory of Networks and Systems, Beer Sheva, Israel, 1983.

9. A. Tannenbaum and P. P. Khargonekar, "On weak pole placement of linear systems depending on parameters," to appear in Proc. 1983 Symposium on the Math. Theory of Networks and Systems, Beer-Sheva, Israel, 1983.

10. A. Tannenbaum, "On a certain class of real algebras which are projective-free," to appear in Archiv der Mathematik.

11. E. W. Kamen, "Stabilization of linear spatially-distributed continuous-time and discrete-time systems," to appear in Recent Advances in Multidimensional Systems Theory, N. K. Bose, ed., D. Reidel, 1984.

## 3.0 Scientific Personnel Supported by the Contract

E. W. Kamen, Principal Investigator

A. Tannenbaum, Research Associate

J. Ferrer, Predoctoral Student.

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